ENDOGENOUS CHOICE OF TRADE INSTRUMENT UNDER UNCERTAINTY

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Abstract: This paper endogenizes the choice between import tariffs and quotas of two policy active countries in a duopsonistic world market. Without uncertainty, import quotas are welfare superior to import tariffs in equilibrium. If two importers can precommit to a type of instrument before deciding the level of the instrument to use in a future period, an import quota equilibrium emerges. We introduce asymmetric risk in the import demand schedule of the two importers. There exists a range of parameters in which a mixed equilibrium emerges; i.e. one country uses a tariff while the other restricts trade with an import quota. Asymmetry in risk creates incentives for countries to use different types of protection. The likelihood that both importers choose a different trade instrument in equilibrium is increasing with the correlation coefficient of the two random shocks.

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1 - Introduction

There exists a large body of literature on the (non) equivalence of trade instruments. Uncertainty and imperfect competition have often been introduced to examine the aforementioned theoretical question. However, researchers have often neglected to endogenize the type of instrument used by policy makers as the strategy space of policy active governments is usually specified exogenously. Some research efforts on endogenous type of protection have originated from the literature on the political economy of trade protection.¹

In this paper, we use a world market structure first introduced by Bergstrom (1982) and later used by Karp and Newbery (1991) and Gervais and Lapan (1998). There exist $N$ policy active importers who behave non-cooperatively to exercise their market power over a certain good. The rest of the world consists of a large number of countries, which behave passively and pursue free trade. We proceed to endogenize the importers’ choice of trade instrument. We propose a two-stage game following Copeland et. al. (1989).²

¹ There exists a considerable theoretical and empirical literature on the political economy of protection. A useful survey of the issues is found in Rodrik (1995).
² Copeland et. al. (1989) compare Nash bargaining games of tariffs and quotas in a bilateral monopoly framework. They show that even if a quota war is Pareto inferior to a tariff war, the two countries will not unanimously agree to ban quotas since the stronger bargaining government will wish to use the threat of quotas in the Nash bargaining game as a strategic tool to influence the outcome.
importers simultaneously precommit to a type of instrument they will use to restrict trade. In the second stage, given the observed type of instrument the other importer has committed to use, each country chooses the level of its trade policy. This type of game can be rationalized as an international trade game where countries agree whether to use tariffs or quotas and then decide the level of trade protection non-cooperatively.

The objective of importers is to capture a potential terms of trade gain at the expense of increased domestic deadweight loss. In a symmetric equilibrium without uncertainty, Gervais and Lapan (1998) have shown that import tariffs are welfare inferior to import quotas in equilibrium. A substantial literature examines the (non)equivalence of tariffs and quotas under uncertainty [Fishelson and Flatters (1975), Dasgupta and Stiglitz (1977), Pelcovits (1977) and Young (1979, 1980)]. It is generally recognized that tariffs are preferred to import quotas under uncertainty since they allow the arbitrage of imports across states of the world while quantitative restrictions at the border fix imports. While the previous papers do not explicitly consider risk preferences, Young and Anderson (1982) compare tariffs and quotas in a general equilibrium framework. The tariff arbitrage also implies that real income fluctuations are greater under a tariff than a quota. If individuals are risk averse, the fluctuations in income reduce the attractiveness of the tariff compared to the quota.

This literature generally suffers from one major flaw. It fails to identify why there is protection in the first place. It has been the usual practice to compare tariffs and quotas under the expected import criterion or some other non-economic objective. The results should not be interpreted as offering an argument for protection per se. One notable exception is found in Lapan and Choi (1988). They provide a ranking of tariffs and quotas under an import-induced externality for a small country facing foreign price uncertainty and domestic production
disturbances. In our paper, trade policy is utilized to equate marginal cost (domestic distortion cost of the policy) and marginal benefit (terms of trade gain) from trade. That is, if a country is large enough to affect the price of the good it buys, it can usually gain by restricting trade below the free trade level. We introduce uncertainty in the production schedules of each importer. We proceed to endogenize the type of trade instrument used by each importer under risk. Hence, we do not restrict the strategy space of importers \textit{a priori}. In a bilateral monopoly framework, Grant and Quiggin (1997) have endogenized the type of tariff used by large countries under uncertainty. Given the structure of the export supply (import demand) functions, the optimal type of tariff in equilibrium vary from a specific tariff to an \textit{ad-valorem} tariff and to a quadratic tariff.

Our paper differs from the existing literature in two major aspects. While Grant and Quiggin have considered only tariff symmetric equilibria, we propose an asymmetric model and allow importers to use a different type of instrument in the equilibrium of the game.\footnote{Our analysis has similarities with the theory of mixed oligopoly. Singh and Vives (1984) and Cheng (1985) study an oligopoly structure where price setting and quantity setting firms co-exist in a risk-free environment.} Second, our market structure does not imply a discontinuity in a player’s payoff in the quota game. In a bilateral monopoly, the payoff to the exporter’s quota leads to a desire to undercut the opponent’s quota whereas no such discontinuity occurs when both importer and exporter use tariffs. When the policy active countries are all importers (or all exporters), neither policy instrument leads to a discontinuity in the welfare function.

This paper is organized as follows. Section 2 solves the equilibrium of the game where importers simultaneously choose the type of trade policy under certainty. Section 3 introduces uncertainty and shows that under asymmetric risk, the endogenous type of protection game may result in a mixed trade instrument regime in equilibrium. Section 4 presents concluding remarks.
2 – Endogenous Choice of Instrument under Certainty

Assume there are two large countries on the world market with purchasing power over the same good \( N = 2 \). We assume two symmetric countries and no uncertainty for the time being. Asymmetric risk will be introduced in the next section. The game with endogenous choice of the type of protection is as follows. In the first stage, the two countries choose either to use an import quota or an import tariff to restrict trade. Given that a country’s choice of a type of instrument is publicly observed, each government simultaneously chooses the level of protection in the second stage. Backward induction is used to find the Subgame Perfect Nash (SPN) equilibrium of the game.

Importing countries are indexed by the subscript \( i = 1, 2 \). We assume preferences and production technology in each country are given by:

\[
U^i(d_i, e_i) = e_i + \sigma_i(d_i); \quad c_i(q_i) + z_i \leq A_i
\]  

where \((d_i, e_i)\) denotes a representative agent’s consumption of the importable and exportable good, and \((q_i, z_i)\) denotes output of the importable and exportable respectively. The second equation in (1) merely represents the production possibility frontier in explicit form, where \( c_i'(q_i) \) is its slope. The quasi-linear specification of preferences implies that the income elasticity of demand for importables is zero.\(^4\) This simplifying assumption is made so that partial equilibrium and general equilibrium analysis are consistent. Using (1) and the trade balance condition, importer \( i \)'s welfare is:

\[
W_i = e_i + \sigma_i(d_i) = (z_i - \bar{p}m_i) + \sigma_i(d_i) = (A_i - c_i(q_i) - \bar{p}m_i) + \sigma_i(d_i); \quad d_i = q_i + m_i
\]  

\(^4\) Throughout the paper we restrict attention to interior solutions.
where \( \bar{p}m_i \) represents exports. For simplicity, we work with a value function defined in terms of imports. Define the function \( V(m_i) \) as: \( V(m_i) = \max_{q_i} \{ \sigma_i(q_i + m_i) - c_i(q_i) \} \). Absent any domestic production policies, the FOC for this problem is given by: \( \sigma_i'(q_i + m_i) - c_i'(q_i) = 0 \) .

A few remarks regarding the notation are in order before we proceed with the analysis. The term \( W_{i}^{ab} \) denotes the welfare of country \( i \) in equilibrium if country 1 uses instrument \( a \) while importer 2 uses instrument \( b \). The two instruments available to importers are an import tariff and import quota, denoted \( \tau \) and \( m \) respectively. Similarly, the expression \( m_{i}^{ab} \) denotes the equilibrium import quantities of country \( i \) if country 1 uses instrument \( a \) while importer 2 uses instrument \( b \). Since we extensively refer to the previous notation in the text, table 1 summarizes the notation used throughout.

Table 1. Notational definitions

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Game</th>
<th>Equilibrium imports and welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country 1 uses ( \tau ), country 2 uses ( \tau )</td>
<td>([\tau, \tau])</td>
<td>(m_{1}^{\tau}, W_{1}^{\tau}) (m_{2}^{\tau}, W_{2}^{\tau})</td>
</tr>
<tr>
<td>Country 1 uses ( m ), country 2 uses ( m )</td>
<td>([m, m])</td>
<td>(m_{1}^{mm}, W_{1}^{mm}) (m_{2}^{mm}, W_{2}^{mm})</td>
</tr>
<tr>
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<td>(m_{1}^{m\tau}, W_{1}^{m\tau}) (m_{2}^{m\tau}, W_{2}^{m\tau})</td>
</tr>
</tbody>
</table>

The quota optimization problem for country \( i \), given country \( j \) uses an import quota, is:

\[
\text{Max}_{m_i} W_i(m_1, m_2) = V_i(m_i) - \bar{p}(m_1 + m_2)m_i
\]

(3)

\(5\) For simplicity, we drop the parameter \( A_i \); nothing is lost by doing so.
where the inverse foreign export supply \( \bar{p}(m_1 + m_2) \) is defined by the world trade equilibrium condition: \( m_1 + m_2 = \bar{S}(\bar{p}) - \bar{D}(\bar{p}) \). Foreigners’ supply and demand functions are respectively \( \bar{S}(\bar{p}) \) and \( \bar{D}(\bar{p}) \). Importers are sufficiently large (in economic terms) to influence the world price at which they trade, whereas exporters are assumed to pursue free trade.\(^6\) Thus, 
\[
(d\bar{p}/dm) = (\bar{S}' - \bar{D}')^{-1} > 0.
\]
The first order condition of the maximization problem in (3) is:
\[
V'_i - \bar{p} - m_i \bar{p}' = 0, \quad i = 1, 2
\]
given country \( i \)'s beliefs that \( j \) uses a quota. Assuming that the welfare function is strictly concave in its own imports, the second order condition for a maximum is satisfied,
\[
V''_i - 2\bar{p}' - m_i \bar{p}'' < 0.
\]
Solving (4) simultaneously for both importers yields the Nash equilibrium \( (m_1^{mm}, m_2^{mm}) \). We assume that import quotas are strategic substitutes; hence: \(-\bar{p}' - m_j \bar{p}'' < 0\).

Therefore, along the reaction function of country \( i \), we have:
\[
\frac{\partial m_i}{\partial m_j} = \frac{-\bar{p}' + m_i \bar{p}''}{V''_i - 2\bar{p}' - m_i \bar{p}''} \in (-1, 0), \quad i = 1, 2; \quad i \neq j
\]
(5)

Suppose now that both countries use a specific tariff. Country \( i \) maximizes (3), assuming country \( j \)'s tariff (not imports) is given. Given \( j \)'s import tariff, \( V'_j(m_j) = \bar{p}(m_i + m_j) + \tau_j \) defines \( m_j(m_i, \tau_j) \), and \( \frac{\partial m_j}{\partial m_i} \bigg|_{\tau_j} = \frac{-\bar{p}'}{(V''_j - \bar{p}')} < 0 \). Given \( j \)'s tariff, there is a one-to-one correspondence between \( i \)'s tariff and imports, and thus country \( i \) is indifferent as to which instrument it uses.\(^7\)

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\(^6\) As usual, this can be rationalized by assuming there are a large number of exporters who behave non-cooperatively, so that no one exporter has significant market power.

\(^7\) Each importer is indifferent as to which instrument it uses to restrict trade. Country \( i \)'s beliefs about what instrument country \( j \) uses is the crucial assumption. For simplicity, we use imports as the choice variable in (6).
This relationship is defined by: \( V'_i(m_i) = \bar{p}(m_i + m_j\langle m_i, \tau_j \rangle) + \tau_i \). Given \( \tau_j \), the optimal imports (or tariff) for \( i \) is given by:

\[
\frac{\partial W_i}{\partial m_i} = V'_i - \bar{p} - m_i \bar{p}' - m_j \bar{p}' \left( \frac{\partial m_j}{\partial m_i} \right) |_{\tau_j} = 0 \rightarrow V'_i - \bar{p} - m_j \bar{p}' \left( \frac{V''_j}{V''_j - \bar{p}'} \right) = 0, \text{ for } i = 1, 2; \text{ for } i \neq j
\]

Equation (6) can be used to determine the reaction functions in tariff space, or the implicit reaction functions in import space (even though tariffs are the strategic variables). Solving the first order conditions in (6) simultaneously for both importers yields the Nash equilibrium \( (m'^{\tau_1}, m'^{\tau_2}) \). Evaluate (6) at the import quota equilibrium \( (m'^{mm}_1, m'^{mm}_2) \) to get:

\[
\frac{\partial W_i}{\partial m_i} \bigg|_{(m'^{mm}_1, m'^{mm}_2)} = -m_i \bar{p}' \left( \frac{\bar{p}'}{V''_j - \bar{p}'} \right) > 0
\]

Equation (7) implies that \( (m'^{\tau_1}, m'^{\tau_2}) > (m'^{mm}_1, m'^{mm}_2) \). Because of the non-cooperative nature of the trade policy game, both countries import more than \( \tau_i \) is collectively optimal in both the symmetric tariff and quota games. Under a strategic quota game, as each country expands its imports, it assumes the imports of other countries remain fixed, so that the increase in the world price only depends upon the foreign export supply elasticity. However, under the tariff game, an increase in imports drives up world prices and reduces imports of other nations, thus lessening the overall impact on the world price. Consequently, the policy-active importers prefer the quota equilibrium since it yields a solution closer to the collusive optimum. The next step is to find the equilibrium policies associated with the mixed instrument game.

Suppose, without loss of generality, that country 1 uses a quota and that country 2 uses a tariff. From country 1’s perspective, \( \tau_2 \) is held fixed when it optimizes. Conversely, from
country 2’s perspective, country 1’s imports are fixed when it optimizes. The equilibrium of the mixed instrument game solves the set of first order conditions:

\[ V'_1 - \bar{p} - m_1 \bar{p} \left( \frac{V''_2}{V'_2 - \bar{p}} \right) = 0, \quad \text{if country 1 believes 2 uses a tariff} \quad (8) \]

\[ V'_2 - \bar{p} - m_2 \bar{p}' = 0, \quad \text{if country 2 believes country 1 uses a quota} \quad (9) \]

Each of these can be used to implicitly define best response functions in either tariff or import space. Solving (8) and (9) simultaneously yields the Nash equilibrium \((m_1^{mc}, m_2^{mc})\).

Evaluating (8) and (9) at \((m_1^{mn}, m_2^{mn})\) gives:

\[ \left. \frac{\partial W_1}{\partial m_1} \right|_{(m_1^{mn}, m_2^{mn})} = -m_1 \bar{p}' \frac{\bar{p}'}{V'_2 - \bar{p}} > 0; \quad \left. \frac{\partial W_2}{\partial m_2} \right|_{(m_1^{mn}, m_2^{mn})} = 0 \quad (10) \]

Similarly, evaluating (8) and (9) at \((m_1^{nr}, m_2^{nr})\) yields:

\[ \left. \frac{\partial W_1}{\partial m_1} \right|_{(m_1^{nr}, m_2^{nr})} = 0; \quad \left. \frac{\partial W_2}{\partial m_2} \right|_{(m_1^{nr}, m_2^{nr})} = \frac{m_2 (\bar{p})^2}{V'_2 - \bar{p}} < 0 \quad (11) \]

**Proposition 1**: Consider two symmetric policy active importers. Import quantities are assumed to be strategic substitutes. Suppose country 1 restricts trade with an import quota while the second importer restricts trade using a tariff. In equilibrium, we have that: (i) \(m_1^{mc} > m_1^{mn}\) and \(m_2^{mc} < m_2^{mn}\); (ii) \(W_2^{mc} < W_2^{mn}\); (iii) \(m_1^{mr} < m_1^{mc}\) and \(m_2^{mr} > m_2^{mc}\); and (iv) \(W_1^{mc} > W_1^{mr}\).

**Proof**: Since country 2 believes country 1 uses an import quota, its best response function (in import space) is the same as in the quota-quota game, and hence the equilibrium for this game must lie on country 2’s reaction function. Country 1’s first order condition, from (10), shows that it will expand imports beyond its equilibrium level in the quota-quota game. This, in
conjunction with the fact the reaction function is downward sloping, yields result (i). Moreover, since we move along country 2’s reaction function to higher output levels for country 1, welfare for country 2 must be lower. Formally, totally differentiate (3) to get:

\[ dW_2 = (V' - \bar{p} - m_2 \bar{p}') dm_2 - m_2 \bar{p}' dm_1 = -m_2 \bar{p}' dm_1 \] (12)

Since \( m_1 \) increases, this implies: \( W_2^{mt} < W_2^{mm} \), proving claim (ii).

From (8), the equilibrium of the mixed \([m,\tau]\) game may also be viewed as being along country 1’s reaction function from the tariff-tariff game (drawn in quantity space). From (11), country 2 will import less than in the tariff-tariff game. This can be viewed as an inward shift of country 2’s reaction function (compared to the tariff-tariff game), and hence the equilibrium is further along country 1’s reaction function (i.e., country 1’s output level increases). Thus, \( m_1^{tt} < m_1^{me} \) and \( m_2^{tt} > m_2^{me} \), proving claim (iii). Since we move down country 1’s reaction function, its welfare is higher in the mixed \([m,\tau]\) game than in the \([\tau,\tau]\) game. Formally,

\[ dW_1|_{RF} = -m_1 \bar{p}' dm_2 > 0 \text{ since } dm_2 > 0. \] Hence, it follows that: \( W_1^{me} > W_1^{	au	au} \). Q.E.D.

Given country \( i \)'s choice of the type and level of trade policy, country \( j \) is indifferent as to which instrument it uses. Thus, a country's choice of a tariff or quota strategy does not change its own reaction function but that of its rival. When country 1 optimizes, it faces a more elastic residual foreign export supply when the other country’s tariff, rather than its imports, are held fixed since, given \( \tau_2 \), increases in \( m_1 \) reduce \( m_2 \). Therefore, country 1 imposes a higher quota when country 2 uses a tariff than when it uses a quota.

Figure 1 explains the intuition behind the results of Proposition 1. The two solid lines represent the reaction function of both countries, given each believes the other uses an import
quota, and thus point A represents the equilibrium of the \([m,m]\) game. Each of the dotted loci represents the best response function when that country believes the other importer uses a tariff as its strategic variable.\(^8\) Note that country \(i\)'s belief that country \(j\) uses a tariff (rather than a quota) causes country \(i\) to increase imports, since it knows country \(j\) will reduce its own imports in response. Thus, country \(i\)'s best response function pivots outward, as compared to the case when it believes country \(j\) uses a quota. Thus, the point \(B\) represents the equilibrium of the symmetric tariff game \((m^\tau_1, m^\tau_2)\). When country 1 uses a quota and country 2 uses a tariff, the appropriate best response functions are the solid locus for country 2 and the dotted locus for country 1, with the corresponding equilibrium at point C. Compared to the symmetric quota game, this may be viewed as an outward shift of country 1’s reaction function, leading to an equilibrium along country 2’s reaction function, with lower imports – and lower welfare – for country 2, and higher imports for country 1. Alternatively, as compared to the symmetric tariff game, the equilibrium occurs along country 1’s (tariff) reaction function, due to the inward shift of country 2’s reaction function. This inward shift of country 2’s reaction function leads to higher welfare for country 1 than in the symmetric \([\tau,\tau]\) game.

Using symmetry and the results of Proposition 1, we have that: \(W^{\text{qm}}_1 < W^{\text{mm}}_1\) and \(W^{\text{qm}}_1 > W^{\text{tt}}_1\). Hence, an import quota is a dominant strategy for country 1. From symmetry, it follows that an import quota is also a dominant strategy for country 2. Suppose we assume that importers in the first stage of the game simultaneously commit to the trade instrument that they will use in the second stage. The pair \((m^{\text{mm}}_1, m^{\text{mm}}_2) \rightarrow (W^{\text{mm}}_1, W^{\text{mm}}_2)\) is a Subgame Perfect Nash (SPN) equilibrium of this game.

\(^8\) For the tariff-tariff game, these response functions are usually drawn in tariff space. However, given the FOC and
3 – Production Uncertainty and the Endogenous Choice of Trade Instrument

In this section, we introduce asymmetric risk in the import demand schedule of importers. In particular, we assume that governments, which possess identical information sets, know the structure, but not the actual level of production costs, when their decisions are made. As we will show below, the introduction of risk may reverse the ranking between the quota and tariff. Additionally, the asymmetry in risk may induce the governments of importing nations to choose different instruments in equilibrium. The sequence of events in the endogenous type of protection game with uncertainty is as follows:

1. The governments of the importing nations simultaneously choose, and commit to, the type of trade restriction (quota or tariff). This decision is publicly observed.
2. Given the irrevocable commitment to the type of trade instrument, both countries choose the level of their trade policy.
3. Production shocks are realized, and commonly observed.
4. Private production, consumption and trade decisions are made and implemented.

We assume, as earlier, that preferences and the structure of technology are identical in both countries. Domestic preferences in each importing country are represented by a quasi-linear utility function: 

\[ U(e_i, d_i) = e_i + \frac{a_i}{b} d_i - \frac{d_i^2}{2b} \]

where \( e_i \) is the numéraire good. The structure of preferences yields a linear domestic demand for the import good: 

\[ d_i = a - b p_i \]

where \( a \) and \( b \) are positive constants and \( p_i \) is the domestic price. Domestic producers of the importable in country \( i \) have the following cost function:

\[ c(q_i, \varepsilon_i) = \frac{q_i^2}{2g} + \frac{c q_i}{g} - \frac{\varepsilon_i q_i}{g} \]

where \( \varepsilon_i \) is a random shock with mean zero and variance \( \sigma_i^2 \). As noted above, the value of the shock \( \varepsilon_i \) is not known when the exogeneity of the specified policy instrument, these response functions can also be drawn in import space.
policy decisions are made, but is known when production decisions are made. A positive realization of $\varepsilon_i$ decreases the marginal and total costs of producers in country $i$. Profit maximization under full information implies: $q_i = -c + gp_i + \varepsilon_i$, where $g > 0$.

Foreigners are passive and follow a free trade policy. The foreign export supply function is: $X = \alpha + \beta \bar{p}$; with $\beta > 0$. The world price is determined according to the equilibrium condition in trade; thus $\bar{p} = [-\alpha + m_2 + m_i] / \beta$ while the domestic price is determined according to the equilibrium on each domestic market: $p_i = [(a + c - \varepsilon_i) - m_i] / (b + g)$. In our partial equilibrium model, welfare of importer $i$ is defined as the sum of consumer surplus, producer surplus and government revenue:

$$W_i = \int_0^q \left( \frac{a - y_i}{b} \right) dy_i - c(q_i) - \bar{p}m_i = \frac{a}{b} d_i - \frac{d_i^2}{2b} - \frac{q_i^2}{2g} + \frac{c}{g} q_i \varepsilon_i q_i - \bar{p}m_i$$

$$= \left[ \frac{2g a - b(c - \varepsilon_i)}{2bg (b + g)} \right]^2 + \frac{2(a + c - \varepsilon_i) m_i - (m_i)^2}{2(b + g)} - \bar{p}m_i$$ (13)

Import quota

First, we restrict the strategy space of both importers to an import quota. Both policy active importers maximize expected welfare in (13) given the other importer’s quota. The equilibrium of the game solves the first order condition: $E[p_i - \bar{p} - m_i \bar{p}'] = 0$ for $i = 1, 2$. Let $\lambda = \beta / 2(b + g)$ be the relative slope of the foreign export supply with respect to the slope of the import demand functions. Hence, an increase (decrease) in $\lambda$ implies an increase (decrease) in

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9 The difference in information sets could be accounted for by the inevitable time lag in making policy.

10 Because of the linearity assumption in our model, we cannot distinguish the production shock from a similar linear shock on the intercept of the demand function. The source of the randomness is really in the intercept of the import demand function. We refer to production uncertainty throughout the text for convenience of exposition.
the elasticity of the residual foreign export supply faced by importer $i$. Solving the two first order conditions simultaneously yields the equilibrium import quota of both importers:

$$m_i^{mm} = \frac{2A}{2(1+\lambda) + 1}$$

(14)

where $A \equiv (a + c)\lambda + \alpha'$ and $\alpha' = \alpha/2$.

**Import tariff**

Assume both importers use a specific tariff $\tau_i$. Imports in country $i$ are defined as the difference between domestic demand and domestic supply at a given price:

$$m_i = (a - c) - (b + g)(\bar{p} + \tau_i) - \varepsilon_i$$

(15)

The world price is determined according to the equilibrium condition $m_i + m_2 = \bar{X}$, which implies:

$$\bar{p} = \frac{2(a - c) - \alpha - (b + g)(\tau_1 + \tau_2) - (\varepsilon_1 + \varepsilon_2)}{\beta + 2(b + g)}.$$

Both importers maximize expected domestic welfare in (13) given the other importer’s tariff. The first order condition is:

$$\partial_{\tau_i} (\partial m_i / \partial \tau_i) - m_i (\partial \bar{p} / \partial \tau_i) = 0.$$  

With appropriate substitutions, the first order condition implicitly defines the reaction function of both importers:

$$E[m_i - \tau_i (b + g)(1 + 2\lambda)] = 0.$$  

Solving simultaneously the last equation for both importers yields the Nash equilibrium tariff. The equilibrium import quantities in each country are found substituting the equilibrium tariff into (15):

$$m_i^{\tau*} = \frac{A(1 + 2\lambda)}{(1 + 4\lambda + 2\lambda^2)} + \frac{\varepsilon_j - (1 + 2\lambda)\varepsilon_i}{2(1+\lambda)}$$

(16)

From (16), import quantities are functions of the random terms; thus $\partial m_i^{\tau*} / \partial \varepsilon_j > 0$ and $\partial m_i^{\tau*} / \partial \varepsilon_i < 0$. A decrease in country $j$’s marginal cost (due to an increase in $\varepsilon_j$) decreases
country \( j \)'s imports, lowers world price, thereby causing imports of country \( i \) to increase.

Rewriting (16), we have:

\[
m_{i}^{\text{tt}} = m_{i}^{\text{mm}} + \left[ \frac{A}{(1+4\lambda + 2\lambda^2)(2(1+\lambda) + 1)} + \frac{\varepsilon_{i} - (1+2\lambda)\varepsilon_{i}}{2(1+\lambda)} \right] \tag{17}
\]

In a deterministic model, equation (17) implies that \( m_{i}^{\text{tt}} > m_{i}^{\text{mm}} \), in accordance with the result in (7). However, when commercial policy is set prior to the resolution of uncertainty, realizations of the random variables (costs) will affect imports under tariffs, but not under quotas. Thus, while expected imports will be larger when both countries use tariffs than when both use quotas, realized imports might – for some realizations – be lower under tariffs.

**Mixed instruments**

Without loss of generality, assume country 1 uses a specific tariff while country 2’s instrument is a quota. Equilibrium on the world market implies:

\[
\bar{p} = \frac{(a+c) + (\alpha - \delta) - (b + g)\tau_{1} + m_{2}}{\beta + b + g}.
\]

The first order condition of country 1’s maximization problem is: 

\[
E\left[ \tau_{1}(\partial m_{i}/\partial \tau_{1}) - m_{i}(\partial \bar{p}/\partial \tau_{1}) \right] = 0.
\]

The previous equation implicitly defines the tariff reaction function of country 1 as a function of the quota of the second policy active importer: 

\[
\left( (b + g)^2 - (\beta + b + g)^2 \right) \tau_{1} = -(a - c)\beta - (b + g)\alpha + (b + g)m_{2}.
\]

Rearranging terms in the tariff reaction function yields:

\[
\tau_{1} = \frac{A}{2\lambda(b + g)(1+\lambda)} - \frac{m_{2}}{4\lambda(b + g)(1+\lambda)} \tag{18}
\]

The first order condition of country 2 is: 

\[
E\left[ p_{2} - \bar{p} - m_{2}/(\bar{S}_{r} - \bar{D}_{r}) \right] = 0.
\]

The quota reaction function of 2 as a function of country 1’s tariff is:
Using (18) and (19), we obtain the Nash equilibrium import quantities in the \([\tau, m]\) game:

\[
m^*_2 = \frac{2A + (b + g)\tau_1}{(2\lambda + 3)} \tag{19}
\]

\[
m^*_2 = \frac{2A[1 + 4\lambda(1 + \lambda)]}{4\lambda(1 + \lambda)(2\lambda + 3) + 1}
\Rightarrow m^*_1 = \frac{16A\lambda(1 + \lambda)}{4\lambda(1 + \lambda)(2\lambda + 3) + 1}(1 + 2\lambda) - \frac{2\lambda\varepsilon_1}{1 + 2\lambda} \tag{20}
\]

**Equilibria of the game**

In the first stage of the game, both policy active importers simultaneously announce the trade instrument they commit to use in the second stage. In order to compare the expected welfare levels associated with each equilibrium, we rewrite the welfare function in terms of imports and take the expectation with respect to the two random terms. Substituting the producers’ cost function, demand schedules and foreign export supply into (13) yields:

\[
E[W_i] = K + \frac{1}{2(b + g)} \left[ \frac{2Am_j}{\lambda} - \frac{(1 + \lambda)}{\lambda} m^2 + \frac{m_j}{\lambda} + \frac{\mu}{(1 - \mu)} \varepsilon_i^2 + 2\varepsilon_i \left( a - \frac{c\mu}{(1 - \mu)} - m_i \right) \right] \tag{22}
\]

where \( K \equiv a^2 / 2b + c^2 / 2g - (a + c)^2 / 2(b + g) \) and \( \mu \equiv b / (b + g) \). Assume the two random shocks are jointly distributed and drawn from a bivariate normal distribution with density function:

\[
f(\varepsilon_1, \varepsilon_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1 - \rho^2}} \exp \left[ -\frac{1}{2(1 - \rho^2)} \begin{bmatrix} \varepsilon_1 & \varepsilon_2 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_1 \sigma_2 & \sigma_2^2 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \right] \tag{23}
\]

From the properties of a bivariate normal distribution, we have that the covariance between two random variables is: \( \sigma_{12} = \rho \sigma_1 \sigma_2 \). For simplicity, write the standard error of \( \varepsilon_2 \) as a function of the standard error of \( \varepsilon_1 \): \( \sigma_2 = \phi \sigma_1 \); thus, \( \sigma_2^2 = \phi^2 \sigma_1^2 \) and \( \sigma_{12} = \rho \phi \sigma_1^2 \). The parameter \( \phi \) measures
the degree of asymmetry in each importer’s production shock: if $\phi$ is greater (lower) than one, country 2 faces greater (lower) risk in production than country 1. Under these assumptions, we can use (14) and (22) to compute the expected welfare levels of the import quota game:

$$W_i^{num} = K + \frac{1}{2(b + g)} \left\{ \frac{4A^2(1 + \lambda)}{\lambda(3 + 2\lambda)^2} + \frac{\mu\sigma_i^2}{(1 - \mu)} \right\}$$

(24)

Similarly, combining (16), (20) and (21), we can substitute back into (22) to compute the expected welfare levels under the various equilibria: $W_i^{rr}$ for $i = 1, 2$; $W_1^{rm}$, $W_1^{mr}$, $W_2^{rm}$ and $W_2^{mr}$. The expected welfare levels under the various trade policy regimes are given in Appendix A.

Consider two policy active importers who simultaneously commit either to use an import tariff or an import quota to restrict trade in the first stage of the game. Their trade instrument commitment is publicly observed. In the second stage, both importers simultaneously set their trade policy level, given their commitment at the beginning of the game. Each importing nation faces a random production disturbance, and neither shock is observed before the choice of the type, or level, of trade policy. Finally, production disturbances are observed and production, consumption and trade are carried out.

**Proposition 2**: Assume the random shocks are distributed according to (23) and $\phi > 1$. (i) There exists a SPN equilibrium to the endogenous type of instrument game in which both importers use an import quota if: $\frac{\sigma_i^2}{A^2} < g_1(\lambda)/\phi^2$; (ii) Both importers enter into a prisoner’s dilemma in the SPN equilibrium in (i) if: $\frac{g_2(\lambda)}{1 + 4\lambda(1 + \lambda) + \phi^2 - 2\rho\phi(1 + 2\lambda)} < \frac{\sigma_i^2}{A^2} < g_1(\lambda)/\phi^2$.

**Proof**: See Appendix A.\(^{11}\)

\(^{11}\) The terms $g_1(\lambda)$ and $g_2(\lambda)$ are defined in the appendix.
The inequality $\sigma_1^2 / A^2 < g_1(\lambda) / \phi^2$ implies that $W_{2_{mm}} > W_{2_{rr}}$; where $g_1(\lambda)$ is a polynomial function of the parameter $\lambda$. Since $\phi > 1$ and $W_{2_{rr}}$ is increasing in $\phi$, it must be that country 1’s best response to country 2 using a quota is to use an import quota as well; hence $W_{1_{mm}} > W_{1_{rr}}$. Therefore, if the other country uses a quota, an importer’s best response is also to use an import quota in the first stage of the game provided the variance in the production shock is not too large.

The second inequality $g_2(\lambda) / \left[ 1 + 4\lambda(1+\lambda) + \phi^2 - 2\rho\phi(1+2\lambda) \right] < \sigma_1^2 / A^2$ implies that: $W_{1_{rr}} > W_{1_{mm}}$; where $g_2(\lambda)$ is a polynomial function of $\lambda$. Moreover, $W_{2_{rr}} > W_{2_{mm}}$ since $W_{2_{rr}} > W_{1_{rr}}$ for $\phi > 1$. Both importers would be better off in terms of expected welfare if they agreed to restrict policy to import tariffs, even if they cannot collusively set the policy level.

Given a sufficiently low variance in the production shocks, quotas are preferred to import tariffs for both countries. As $\rho$ increases (decreases), the likelihood that both importers enter a prisoner’s dilemma in equilibrium decreases (increases). Expected welfare in the $[\tau, \tau]$ game is decreasing in $\rho$ for both countries. Intuitively, each country prefers imports to be negatively correlated, so that when its imports are above normal (due to a negative productivity shock), world price does not increase as much due to the offsetting reduction in the other county’s imports (comparable statements apply when imports are below normal). Numerical simulations at the end of this section highlight the role of the risk parameters $(\rho, \phi)$ in the equilibria of the game.
Proposition 3: Assume the random shocks are distributed according to (23) and $\phi > 1$. The SPN equilibrium of the game entails country 1 using an import quota and country 2 using an import tariff if: $g_1(\lambda)/\phi^2 < \sigma_1^2/A^2 < \frac{g_3(\lambda)}{(1+2\lambda)^2 + \phi^2 - 2\phi(1+2\lambda)}$.

Proof: See Appendix A.

Given $\phi > 1$, country 2 faces greater risk than country 1. If $\sigma_1^2/A^2$ is located in the interval specified in Proposition 3, country 2’s best response to an import quota is to use an import tariff, while country 1’s best response to an import tariff is to use an import quota. Since country 1 faces relatively little risk, it prefers a quota since this tends to reduce country 2’s imports. On the other hand, due to its relative high variance in production, Country 2’s best response is a tariff that allows imports to vary across the states of the world.

A number of observations can be made regarding the proposition above. First, an increase (decrease) in the correlation coefficient $\rho$ increases (decreases) the likelihood of an asymmetric equilibrium in the trade instrument. The coefficient $\rho$ is negatively correlated with the expected welfare in the $[\tau, \tau]$ game. Hence, an increase in $\rho$ decreases the incentive of country 1 to respond to country 2’s import tariff with a tariff, while it does not affect country 2’s best response to country 1’s quota. Second, an increase (decrease) in the asymmetry parameter is sufficient to increase (decrease) the likelihood of a mixed trade policy regime in equilibrium if $\phi < \rho(1+2\lambda)$. An increase in country 2’s relative production risk with respect to the production risk in nation 1 has two effects. The direct effect encourages country 2 to use a tariff instead of a quota since it is facing a larger risk. The indirect effect works in the opposite direction if the correlation coefficient is sufficiently small. A positive correlation between the two shocks implies a high probability that country 1’s random shock will also be large and thus induces
importer 1 to use an import tariff and reduces the attractiveness of a tariff for country 2. Thus, the impact on the equilibrium of the game of an increase in asymmetry is ambiguous. Finally, note that the interval $\frac{g_1(\lambda)}{\phi^2} < \sigma_1^2 / A^2 < \frac{g_1(\lambda)}{(1+2\lambda)^2 + \phi^2 - 2\rho \phi (1+2\lambda)}$ may be empty, implying that only symmetric (in policy space) equilibria exist.

**Proposition 4**: Assume the random shocks are distributed according to (23) and $\phi > 1$. The SPN equilibrium of the game entails both importers choosing an import tariff if:

$$\sigma_1^2 / A^2 > \frac{g_1(\lambda)}{(1+2\lambda)^2 + \phi^2 - 2\rho \phi (1+2\lambda)}.$$  

**Proof**: See Appendix A.

The equilibrium of the game may not be unique since the condition

$$\sigma_1^2 / A^2 > \frac{g_1(\lambda)}{(1+2\lambda)^2 + \phi^2 - 2\rho \phi (1+2\lambda)}$$

does not imply $[\sigma_1^2 / A] > [g_1(\lambda)/\phi^2]$. However, from Proposition 2, the equilibrium where both countries use an import quota will be Pareto dominated by the SPN equilibrium in which both countries use tariffs if $\lambda > 0.487$.  

Using numerical simulation tools and Propositions 2 through 4, it is possible to gauge the impacts of the risk parameters $\rho$ and $\phi$ on the equilibria of the game. Figure 2 presents the impact of the asymmetric production risk parameter on the various equilibria of the game. For any combination of the relative production shock variability ($\sigma_1^2 / A$) and the parameter $\phi$ located below the solid bold line, the equilibrium of the game is for both countries to use an import quota.  

Regions A, B and C represent these admissible combinations. The likelihood

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12 It can be shown numerically that $g_1(\lambda) < g_3(\lambda)$ if $\lambda > 0.48701$. All programs to replicate the numerical results in this paper can be obtained from the authors upon request.

13 The values of the parameters $\rho$ and $\lambda$ are both fixed at 0.5 in figure 2; *i.e.* the slopes of foreigners’ export supply

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that an import quota regime emerges in equilibrium is decreasing in $\phi$. Any combination of $\sigma_1^2/A$ and $\phi$ located above the dotted line implies that the tariff equilibrium is preferred to the import equilibrium. Importers enter into a prisoners’ dilemma in regions B and C since the import quota equilibrium emerges while both countries would prefer to simultaneously use import tariffs.

Finally, in the regions delimited by E and F, any combination of $\sigma_1^2/A$ and $\phi$ yields a mixed trade policy regime in equilibrium. In these two regions, country 1 uses an import quota while country 2 uses an import tariff. Graphically, the area delimited by E and F is increasing for small values of $\phi$ and decreasing for larger values of $\phi$, *ceteris paribus*. Recall from the discussion following the third proposition that a sufficient condition for an increase in $\phi$ to increase the likelihood of an asymmetric equilibrium is that $\phi < \rho (1+2\lambda)$. Since both the parameters $\rho$ and $\lambda$ are equal to 0.5, the inequality $\phi < \rho (1+2\lambda)$ can not hold.

Figure 3 presents the impacts of the correlation coefficient on the equilibria of the game. Any combination of the relative variance in the production shock ($\sigma_1^2/A$) and the parameter $\rho$ located below the solid bold line implies that both countries use an import quota (regions A, B and C). Any combination of the relative variance in the production shocks ($\sigma_1^2/A$) and the parameter $\rho$ located above the thin solid line entails an equilibrium where both countries use an import tariff (designated by the areas A and D). Hence, there exist two SPN equilibria in region A. The likelihood that both countries use import tariffs is decreasing in $\rho$. Finally, any combination of $\sigma_1^2/A$ and $\rho$ located in regions E and F yields a mixed trade policy equilibrium.

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14 The values of $\phi$ and $\lambda$ in figure 3 are fixed at 2 and 0.5 respectively; i.e. the slope of foreigners export supply and importers’ demand are equal and country 2 is facing twice as much production risk as country 1.
In this example, this can only occur for sufficiently large values of the correlation coefficient. If the shocks are negatively correlated, both importers prefer to use import tariffs.

4 - Conclusion

This paper has endogenized the strategy space of two policy active importers who face endogenous terms of trade while foreign exporters pursue a free trade policy. In a risk-free environment, restricting trade using an import quota is a dominant strategy for both symmetric importers. We introduce uncertainty through the import demand schedule in each importing nation. In order to compare the different trade instrument regimes, trade policy decisions are carried out in two distinct stages. First, each government commits to either use an import quota or an import tariff to restrict trade. This stage may be viewed as setting the rules for international agreements. In the second stage, importers non-cooperatively determine the actual level of protection.

A sufficiently high variance in the random shock reverses the ranking between quotas and tariffs since tariffs permit imports to vary across the states of the world. The value of allowing imports to vary with the production shocks must be offset against the fact that tariffs induce larger expected imports (as the residual foreign export supply faced by the other importer appears more elastic). If the two importers face asymmetric shocks, there exists a range of parameter values for which the equilibrium entails importers choosing different trade policy instruments. The importer facing the larger risk uses an import tariff while the other importer chooses to restrict imports with a quota. Moreover, we show that, given the level of risk, the likelihood that importers choose different trade instruments increases with the correlation coefficient of the shocks.
We have shown that asymmetry in production risk creates incentives for policy active importers to use different instruments to restrict trade. While we examined a simultaneous trade game, an interesting extension to our model would be to examine the impact of uncertainty on trade policy leadership. Suppose a structure where production decisions are made before consumption decisions (e.g. in agricultural markets with spring planting and fall harvest). Each policy active importer could irrevocably set its trade policy before production decisions are made and uncertainty in production resolved, or delay its decision until after firms’ decisions are made and uncertainty resolved. Given a degree of asymmetry in both importers’ production schedules, a country may prefer to set its trade policy before production decisions are made and thus potentially obtain a strategic leadership advantage if the variance of its production shock is small compared to the other country. However, the potential first-move advantage must be weighted against the benefit of delaying its trade policy decision if additional information is revealed once firms have moved.
5 – References


Appendix A

Using (16) to substitute into (22), we have:

\[
W_1^{et} = K + \frac{1}{2(b + g)} \left[ \frac{A^2(1 + 2\lambda)(3 + 2\lambda)}{1 + 4\lambda + 2\lambda^2} + \frac{\sigma_i^2\left[(1 + 2\lambda)^2 + \phi^2 - 2\rho\phi(1 + 2\lambda)\right]}{4(1 + \lambda)^2} + \frac{\mu\sigma_i^2}{(1 - \mu)} \right] \quad (A.1)
\]

\[
W_2^{et} = K + \frac{1}{2(b + g)} \left[ \frac{A^2(1 + 2\lambda)(3 + 2\lambda)}{1 + 4\lambda + 2\lambda^2} + \frac{\sigma_i^2\left[(1 + 2\lambda)^2 + \phi^2 + 1 - 2\rho\phi(1 + 2\lambda)\right]}{4(1 + \lambda)^2} + \frac{\mu\phi^2\sigma_i^2}{(1 - \mu)} \right] \quad (A.2)
\]

The impact of a change in \( \phi \) on the expected welfare of both countries under the tariff regime is indeterminate. An increase in country 2’s production risks increases expected welfare of both countries if the correlation coefficient between the two shocks is not too large. Expected welfare under a tariff regime is also a decreasing function of the correlation coefficient.

Using equations (20) and (21) to substitute into (22), we have:

\[
W_1^{et} = K + \frac{1}{2(b + g)} \left\{ \frac{4A^2(1 + 2\lambda)(18\lambda + 28\lambda^2 + 8\lambda^3 - 1)}{[1 + 12\lambda + 20\lambda^2 + 8\lambda^3]^2} + \frac{\mu\sigma_i^2}{(1 - \mu)} \right\} \quad (A.3)
\]

\[
W_1^{et} = K + \frac{1}{2(b + g)} \left\{ \frac{512A^2\lambda^2(1 + \lambda)^3}{(1 + 2\lambda)^2[1 + 12\lambda + 20\lambda^2 + 8\lambda^3]^2} + \frac{4\lambda^2\sigma_i^2}{(1 + 2\lambda)^2} + \frac{\mu\sigma_i^2}{(1 - \mu)} \right\} \quad (A.4)
\]

\[
W_2^{et} = K + \frac{1}{2(b + g)} \left\{ \frac{4A^2(1 + 2\lambda)(18\lambda + 28\lambda^2 + 8\lambda^3 - 1)}{[1 + 12\lambda + 20\lambda^2 + 8\lambda^3]^2} + \frac{\mu\phi^2\sigma_i^2}{(1 - \mu)} \right\} \quad (A.5)
\]

\[
W_2^{et} = K + \frac{1}{2(b + g)} \left\{ \frac{512A^2\lambda^2(1 + \lambda)^3}{(1 + 2\lambda)^2[1 + 12\lambda + 20\lambda^2 + 8\lambda^3]^2} + \frac{4\lambda^2\phi^2\sigma_i^2}{(1 + 2\lambda)^2} + \frac{\mu\phi^2\sigma_i^2}{(1 - \mu)} \right\} \quad (A.6)
\]

Note that an increase in the asymmetry parameter \( \phi \) increases the expected welfare levels \( W_2^{et} \) and \( W_2^{et} \).
Proof of Proposition 2:

Combining (24) and (A.6), we obtain that $W_{2}^{mm} - W_{2}^{mt} > 0$ if $\sigma_{i}^2/A^2 < g_{1}(\lambda)/\phi^2$; where
$$g_{1}(\lambda) \equiv \frac{(1+\lambda)(1+2\lambda)^2}{\lambda^3} \left[ \frac{1}{(3+2\lambda)^2} - \frac{128\lambda^3(1+\lambda)^2}{(1+2\lambda)^2(1+12\lambda+20\lambda^2+8\lambda^3)^2} \right]$$ under the condition of claim (ii). Since $\phi > 1$, it must be the case that $W_{i}^{mm} > W_{i}^{tm}$ if $\sigma_{i}^2/A^2 < g_{1}(\lambda)/\phi^2$. Therefore, both countries’ best response to the other nation using an import quota is to choose an import quota in the first stage of the game; proving claim (i). Using (24) and (A.1), the inequality
$$g_{2}(\lambda) \equiv 4(1+\lambda)^2/\lambda(3+2\lambda)^2(1+4\lambda+2\lambda^2)^2$$ implies that $W_{2}^{rr} > W_{2}^{mm}$; where
$$g_{2}(\lambda) \equiv \frac{4(1+\lambda)^2(4+9\lambda+4\lambda^2)}{\lambda(3+2\lambda)^2(1+4\lambda+2\lambda^2)^2}.$$ Comparing (A.1) and (A.2), we have that $W_{2}^{rr} > W_{2}^{rr}$ for $\phi > 1$ and thus $W_{2}^{rr} > W_{2}^{mm}$. Since under the conditions of claim (i), $\sigma_{i}^2/A^2 < g_{1}(\lambda)/\phi^2$, the pair of actions $(m_{1}^{mm}, m_{2}^{mm})$ is still a SPN equilibrium of the game. However, both importers would be better off in terms of expected welfare if they collude and use import tariffs since $W_{1}^{rr} > W_{1}^{mm}$ and $W_{2}^{rr} > W_{2}^{mm}$. Q.E.D.

Proof of Proposition 3:

From claim (i) in proposition 2, $g_{1}(\lambda)/\phi^2 < \sigma_{i}^2/A^2$ implies $W_{2}^{mt} > W_{2}^{mm}$. Combining (A.1) and (A.3) implies $W_{1}^{rr} < W_{2}^{mm}$ if $\sigma_{i}^2/A^2 < \frac{g_{3}(\lambda)}{(1+2\lambda)^2+\phi^2-2\rho\phi(1+2\lambda)}$; where
$$g_{3}(\lambda) \equiv 4(1+\lambda)^2(1+2\lambda) \left[ \frac{4(-1+18\lambda+28\lambda^2+8\lambda^3)}{(1+12\lambda+20\lambda^2+8\lambda^3)^2} - \frac{(3+2\lambda)}{(1+4\lambda+2\lambda^2)^2} \right].$$ Hence, using a quota is country 1’s best response given country 2 uses a tariff. Given that country 1 uses an import quota, $W_{2}^{mt} > W_{2}^{mm}$ implies country 2’s best response is to use an import tariff. Q.E.D.
Proof of Proposition 4:

From proposition 3, the inequality \( \frac{\sigma_i^2}{A^2} > g_3(\lambda) \left[ \frac{(1 + 2\lambda)^2 + \phi^2 - 2\rho\phi(1 + 2\lambda)}{} \right] \) implies that \( W_i^{it} > W_i^{in} \). It follows immediately that using an import tariff is country 1’s best response to country 2 using an import tariff. Since \( \phi > 1 \), using a tariff must be country 2’s best response to country 1 committing to use a tariff. Q.E.D.
Figure 1. Importers’ reaction functions and equilibria of the import quota game

Figure 2. Impact of the asymmetric production risk parameter on the various equilibria of the endogenous type of protection game
Figure 3. Impact of the production shocks correlation coefficient on the various equilibria of the endogenous type of protection game