1. Introduction

One application of the models studied in this course that will be pursued throughout is money. The purpose is two-fold: It provides an introduction to the key models of money and it illustrates how the methods we develop can be used.

Classical economics postulates that “money is a veil,” which is to say it does not have real effects. Not everyone believes this, but even those who do face a fundamental question: Why and when is money valued in equilibrium? This is not obvious. After all, if anyone handed you a piece of green paper with a picture of a dead president, you would presumably reluctant to hand over anything valuable in return. So what is special about these bits of green paper? The challenge is particularly severe, given that money does not pay interest (“rate of return dominance”). Much of monetary economics simply assumes that people need money for certain transactions, but that approach has problems (Wallace 1997).

From the point of view of OLG models there is an additional question: Can money alleviate dynamic inefficiency? Consider a standard endowment economy with two-period households. Without money, there cannot be any trade in equilibrium. Therefore, the allocation is autarky: \( c^y = e_1 \) and \( c^o = e_2 \). As we have seen, this may not be Pareto optimal. If the marginal rate of transformation is high, \( u'(c^o)/u'(c^y) > 1 + n \), a transfer from current young to current old would be a Pareto improvement. Giving money to the old allows them to trade with the young. Does this mean that dynamic inefficiency goes away?
2. An OLG Model of Money

Consider a standard two period OLG model without production or bonds. The population grows at rate \( n \). Money is introduced as follows. In period 1, the initial old are given \( M \) bits of green paper, each bearing a picture of a dead president and inscribed “One U.S. dollar.” In every subsequent period, the government prints additional paper and hands it to the current old in proportion to the quantity of paper they chose when young. Effectively, money pays (nominal) interest if held from young to old age. This assumption is of course not realistic; the extension to the case where money does not pay interest is discussed in BF, p. 162. The money growth rate is constant:

\[
M_{t+1} / M_t = 1 + \theta .
\]

It would seem that there is no chance that money could be valued in this economy. Suppose I were to follow the same policy of the government and hand out bits of green paper – would you sell me your house for it? (If yes, please let me know asap!). One of the key insights from this model is: the only reason why money is valued is the belief that it will still be valued in the future when the household wants to obtain goods in return for his money holdings. In other words: money is a bubble. Money being valued is merely a self-fulfilling expectation. It’s fragile. Should expectations ever change, the value of money could evaporate. There are of course plenty of historical examples of this (hyperinflation episodes).

2.1 Household

Except for the fact that bonds are replaced by money as the only asset households can hold, the household problem is unchanged. Preferences are \( u(c^y_t, c^o_{t+1}) \). The household receives endowments \( e_1, e_2 \) of (perishable) goods. The price of goods in period \( t \) is \( P_t \). The budget constraints are therefore

\[
e_1 - c^y_t = x_t, \quad P_{t+1} (c^o_{t+1} - e_2) = x_t (1 + \theta) P_t .
\]

The young budget constraint is written in real terms, the old one in nominal terms. The amount of nominal money demanded by a young household is \( P_t x_t \). When old, the household receives money transfers of \( \theta P_t x_t \). Total wealth is then used to buy consumption at price \( P_{t+1} \). The lifetime budget constraint is:

\[
e_1 - c^y_t = \frac{c^o_{t+1} - e_2}{(1 + \theta) P_t / P_{t+1}} .
\]

Note that money acts exactly like a bond that pays gross interest \( R_{t+1} = (1 + \theta) P_t / P_{t+1} \). Set up a Lagrangean

\[
\Gamma = u(c^y_t, c^o_{t+1}) + \lambda_t \left[ e_1 - c^y_t + \frac{e_2 - c^o_{t+1}}{R_{t+1}} \right] .
\]
The FOCs are well-known: $u_1(t) = \lambda_t$, $u_2(t) = \lambda_t / R_{t+1}$. These can be combined into an Euler equation, $u_1(t) = R_{t+1} u_2(t)$. A solution to the household problem is a triple $(c^y_t, c^o_{t+1}, x_t)$ which satisfies the Euler equation and the two budget constraints. Optimal behavior can be characterized by a savings function (which is now a money demand function)

$$x_t = s(R_{t+1}, e_1, e_2).$$

A solution to the household problem is a vector $(c^y_t, c^o_{t+1}, x_t)$ that satisfy 2 budget constraints and the Euler equation. **It is important to realize that the household problem is exactly the same as the one with one period bonds.** This is very general in the models we study here. Since there is no uncertainty, all assets are equivalent to one period bonds.

3. Equilibrium

The **government** is simply described by a money growth rule: $M_{t+1} / M_t = 1 + \theta$.

**Market clearing:** Money supply is exogenous ($M_t$). Money demand is $x_t$, per capita in real terms. If the population size is $N_t$, then

$$M_t = N_t P_t x_t \quad \text{or} \quad m_t = M_t / (N_t P_t) = s(R_{t+1}),$$

where $m$ denotes real, money balances per young household. The goods market clearing condition is $e_1 + e_2 / (1 + n) = c^y_t + c^o_t / (1 + n)$, which is, of course, the same as in the model without money.

To verify that the money market clearing condition together with the household budget constraint implies goods market clearing (**Walras law**), use the young budget constraint of generation $t$ together with the old one of generation $t-1$: $e_1 - c^y_t = x_t$ and $c^o_t - e_2 = x_{t-1} (1 + \theta) P_{t-1} / P_t$. The money market clearing condition provides the link between $x_{t-1}$ and $x_t$:

$$x_t = M_t / (N_t P_t)$$

$$= (1 + \theta) M_{t-1} / [P_t N_{t-1} (1 + n)]$$

$$= (1 + \theta) x_{t-1} (P_{t-1} / P_t) / (1 + n)$$

Therefore, $x_t (1 + n) = (e_1 - c^y_t)(1 + n) = c^o_t - e_2$, which implies the goods market clearing condition.

**Definition:** An **equilibrium** is a sequence $(c^y_t, c^o_t, x_t, P_t, M_t)$ such that

1. $M_t$ obeys the money growth equation. [1 eqn]
2. Markets clear. [2 eqn, one redundant]
3. The household chooses consumption and money holdings optimally. [3 eqn]

We have 5 (independent) equations in 5 unknowns (per period).
4. Characterizing Equilibrium

As in the case of the economy with capital, we can characterize equilibrium by finding a difference equation for the economy’s state variables. These are the money stock and the price level. However, nominal variables play no role in our economy. Therefore, we only need to worry about real money balances as the state variable.

The obvious place to look for a difference equation in \( m_t \) is the money market clearing condition, 

\[
m_t = s(R_{t+1}).
\]

To characterize the path of \( m \), we need to substitute out \( R_{t+1} \). Since \( R_{t+1} = (1 + \theta) P_t / P_{t+1} \), we need an expression for inflation. From

\[
\frac{M_{t+1}}{M_t} = \frac{m_{t+1}}{m_t} \frac{P_{t+1}}{P_t} \frac{N_{t+1}}{N_t}
\]

we have

\[
R_{t+1} = (1 + \theta) P_t / P_{t+1} = (1 + n) m_{t+1} / m_t
\]

\[
\Rightarrow m_t = s((1 + n) m_{t+1} / m_t)
\]

This is a difference equation that completely determines the path for \( m \) (together with a boundary condition). But what does it look like?

Now comes the trick: the “offer curve.” Warning: This is unusual and somewhat hard to grasp (see BF for an exposition). The key idea is to use the household’s intertemporal consumption allocation to figure out how money evolves over time.

Let’s first think about the household’s consumption choice, without any obvious motivation. The lifetime budget constraint tells us that the household faces an intertemporal budget constraint with a slope of \( R_{t+1} \). We next plot how the household’s consumption choices at both dates vary with the interest rate. This is done in the left panel of Figure 1. Each point on the offer curve is obtained as the tangency of an indifference curve with a budget line that has slope \(-R_{t+1}\) and goes through the endowment point.

Thinking about the consumption allocation allows us to place restrictions on the shape of the offer curve. The trouble is that there are income and substitution effects of changing \( R \). For very low levels of \( R \) the household probably wants to borrow. We don’t need to worry about that case since the household cannot do that in this model.
If we start with the interest rate where the household just eats his endowment and then gradually raise it, the household will first reduce $c^y$ because substitution effects dominate. In that region, $s$ is increasing in $R$ and so is obviously $R s$. We therefore know that the offer curve is undefined for too low values of $R$ and downward sloping at least initially for higher levels of $R$. It may eventually bend backwards, but it can intersect each line through the endowment point only exactly once (the solution to the household problem is unique).

Next we relate the offer curve to money. This works because the young budget constraint implies that young consumption is related to real money demand in $t$ as $m_t = s(R_{t+1}) = e_1 - c^y_t$. Something very similar holds for the old: $(1 + n) m_{t+1} = R_{t+1} s(R_{t+1}) = c^p_{t+1} - e_2$. What happens is very clear, if we abstract from population growth for a moment. Young savings constitute real money demand in $t$. Old dissaving at $t+1$ becomes the money supply at $t+1$, which in equilibrium must be purchased by the new young. Hence, if the household wants to postpone a large part of its consumption, then $m_{t+1}$ must be large relative to $m_t$. We can therefore think of the offer curve as describing a relationship $(1 + n) m_{t+1} = F(m_t)$. Here it is tempting to say: “$F$ must be linear because $F(m_t) = R_{t+1} m_t$.” But that is not useful – it merely recovers the definition of $R$.

The right panel of Figure 1 moves the endowment point to the origin and shows a mirror image of the first graph. Since every point on the offer curve is constructed one the budget line for a distinct $R$, the offer curve intersects every ray through the origin exactly once. The horizontal axis now shows $m_t$ and the vertical one shows $(1 + n) m_{t+1}$. In equilibrium, $s = m$ and therefore we can read off $R s = (1 + n) m_{t+1}$ from the offer curve. Using a ray through the origin with slope $(1 + n)$ allows us to find $m_{t+1}$, and so on.

It follows directly that there is one monetary steady state (intersection of offer curve and ray through origin) which is unstable. If the economy is perturbed away from the steady state, the money stock either collapses to zero or blows up to infinity. This happens through inflation or deflation – recall that nominal $M$ grows at a fixed rate.

What are the properties of the steady state? Per capita real money balances, $m$, are constant over time. Since the gross rate of return on money is given by $R_{t+1} = (1 + \theta) P_1 / P_{t+1} = (1 + n) m_{t+1} / m_t$, it must equal $1 + n$ in steady state. Hence, the net return is negative if $n < 0$. The steady state inflation rate is given by $P_{t+1} / P_t = (1 + \theta) / (1 + n)$, which may also be negative.
Offer curve

Figure 1

We can now trace out the **evolution** of the money stock over time. First, how is the initial money stock determined? The answer is: it isn’t. Nothing guarantees that money is even valued in equilibrium. If money is valued, it is only because agents expect that it will be valued in the future. But the equilibrium with \( m_0 = 0 \) (which means \( P_0 = \infty \)) is a perfectly valid equilibrium. But it gets worse: any \( m_0 \) in a certain range yields an equilibrium — there is a continuum of equilibrium paths.

What range of \( m_0 \) values is acceptable? Consider first the case where \( m_0 \) is to the right of the steady state. Such a path would have **deflation**. But such a path is actually not feasible because eventually real money balances would exceed total output. Consider next the case where \( m_0 \) is less than the steady state value. On this **inflationary** path, \( R \) falls over time. Therefore, the inflation rate must be accelerating: the only way to induce people not to save more is to lower the rate of return. The bizarre implication is that money isn’t valued asymptotically (\( P \to \infty \)).

If the money stock is exactly equal to the steady state level, people save exactly the right amount to hold the money stock constant over time. That is, the steady state rate of return must be \( n \). Given the fixed nominal rate of money growth, this requires \( 1 + n = (1 + \theta)/(1 + \pi) \) or approximately \( n = \theta - \pi \).

But now suppose \( m_t \) is a bit below the steady state. Then the equilibrium rate of return must be below \( n \); otherwise people would demand too much money. But that means the inflation rate must be higher than the one that maintains a constant stock of money. So next period \( m \) will be even smaller and the required \( \pi \) even larger. This positive feedback occurs forever, shrinking the money stock at an ever increasing rate.

### 4.1 Dynamic Efficiency

How money relates to dynamic efficiency turns out to depend on the slope of the offer curve at its origin. There are two cases. In the **Samuelson case**, the slope of the offer curve at the origin is less.
than \((1+n)\). The figure above implicitly assumes this case. In the **Classical case**, the slope is greater than \((1+n)\).

In the **Samuelson case** the non-monetary economy is dynamically inefficient. To see this, note that the interest rate in the non-monetary economy is determined by the Euler equation

\[
\beta(1 + r_{t+1})u'(c^0_{t+1}) = u'(c^1_t) .
\]

But in this economy, there is no trade. Therefore,

\[
1 + r = u'(e_1)/[\beta u'(e_2)] .
\]

This means that the interest rate is given by the slope of the indifference curve that goes through the endowment point. The economy is dynamically inefficient, if the interest rate is lower than \(n\), which exactly means that the offer curve is flatter than the \((1+n)\) line.

In the **Classical case** there is no intersection of the offer curve with the \((1+n)\) line (except at zero, of course) and a **monetary equilibrium does not exist** — money is never valued in equilibrium. At the same time the non-monetary economy is dynamically efficient. The main result is therefore:

| Money is valued in equilibrium only in an economy that would be dynamically inefficient without money. |

Note further that the monetary steady state is dynamically **efficient**. The interest rate equals the population growth rate. It is therefore not possible to make households better off by transferring resources from young to old or vice versa. Moreover, the money growth rate does not affect the steady state allocation — money is **super-neutral**. This would be quite different, if money did not pay interest (see BF, chapter 4).

**To summarize**: Starting from a dynamically inefficient non-monetary economy, giving people money results in a dynamically efficient steady state (*if* money is ever valued and *if* the steady state is ever reached). But if the economy is dynamically efficient without money, money will never be valued in equilibrium.

### 5. Is this a good theory of money?

The bits of green paper in this model are not really money. They have one important feature of money: they are intrinsically worthless. But they don’t have the second feature that makes money interesting: rate of **return dominance**. This refers to the fact that the nominal return on money in the real world is zero and is therefore dominated by the returns paid by other assets (such as t-bills).

The theory does accomplish one important task: it highlights how money might be valued just because everybody expects it to be valued tomorrow. Essentially, money can be a bubble: it has no intrinsic value, but is accepted by everyone purely because of these expectations. The theory also highlights how fragile the role of money in such an environment is.
The theory fails miserably, however, when it comes to explaining why anyone would hold money. If one adds another asset to the economy, money will have to pay the same real rate of return if it is to be valued. It is more challenging (and still an open issue) to answer why money would be valued in equilibrium, if there are other assets that pay higher rates of return (Wallace 1997).

6. References

CM 4 has material on money, but better references are: SLj ch. 8, BF ch. 4.1, 5.4; MW ch. 10


An interesting extension (BF p. 162-3) is the case where money does not pay interest. The old simply receive a lump-sum transfer of money in each period. In that case, it can be shown that money is not *superneutral.* That is, the rate of money growth affects the allocation. The monetary steady state is also no longer Pareto-optimal.