1. (Simplified version of Heckscher-Ohlin model). Consider a country which can produce two goods: cloth (C) and food (F) using two inputs: labor (L) and land (T). Production of each good requires inputs to be used in fixed proportions as follows (these are called Leontief technologies):

**To produce cloth:** 2 units of labor and 1 unit of land are required for each unit of cloth.
**To produce food:** 1 unit of labor and 2 units of land are required for each unit of food.

Let $\bar{L}, \bar{T}$ represent the total amount of labor and land available in the economy, let $P_c, P_f$ denote the prices of output and let $W, R$ denote the prices of labor and land respectively.

**a)** Find production costs and hence output price (price=marginal cost) for each good in terms of factor prices ($W, R$). **(Hint:** To produce $Q_c$ units of cloth requires $(2Q_c)$ workers and $(Q_c)$ units of land so total costs are: $(2WQ_c + RQ_c)$. Similar reasoning applies to food).  

i. Use your answers to solve for inputs prices in terms of output prices. How will an increase in the price of cloth affect factor prices?

**b)** Find the production possibility frontier (ppf) for this economy and sketch it. **(Hint:** it is defined by two linear inequalities that state labor (land) used cannot exceed supply. Use the hint from part (a) and also note that producing $Q_f$ units of food requires $(Q_f)$ workers and $(2Q_f)$ units of land.).  

i. Show how an increase in the supply of labor shifts the ppf and the production point where inputs are fully used (in this simple version, there is a unique production point that represents full employment of both inputs).

**c)** Assume two countries (the US and Mexico) have identical tastes and technology, but the U.S. has more land and Mexico has more labor. Assuming the relative demand (ratio of demand for cloth to demand for food) is independent of income, **discuss how autarky goods prices and factor prices differ between the two countries**, then discuss how trade affect factor prices in each country. Will factor prices be equalized between the two countries?

**d)** Modify the above model by assuming US productivity in both sectors doubles. Thus, in the US:

**To produce cloth:** 1 unit of labor and 0.5 units of land are required for each unit of cloth.
**To produce food:** 0.5 units of labor and 1 unit of land are required for each unit of food.

Mexican technology remains unchanged. Viewed in a Ricardian context, the US has an absolute advantage in both goods but a comparative advantage in neither good (due to technology).

i. Find how this US productivity change affects its autarky output prices and factor prices.

ii. Assuming free trade between the US and Mexico, will free trade equalize factor prices? Will trade eliminate the pressure for factor migration?
2. (Labor migration, Ch. 5) The simplest model used for analyzing labor migration is one with only one good and with labor as the only (variable) input. To illustrate, assume food (the only output) is produced using both labor and land according to the following production function:

\[
Q_f^{\text{US}} = 6 \left( \frac{T_f^{\text{US}}}{L_f^{\text{US}}} \right)^{2/3}
\]

where \( T_f^{\text{US}} \) is US land input and \( L_f^{\text{US}} \) is labor input in the US.

The amount of land in the U.S. is fixed at \( T_f^{\text{US}} = 200 \). Labor demand is found by equating the marginal value product of labor to the wage rate (\( W \)). For simplicity, let \( P_f = 1 \) so labor demand is:

\[
MPL = \frac{\partial Q_f^{\text{US}}}{\partial L_f^{\text{US}}} = \frac{4}{3} \left( \frac{T_f^{\text{US}}}{L_f^{\text{US}}} \right)^{1/3} \rightarrow L_f^{\text{US}} = W \cdot \left( L_f^{\text{US}} \right)^d = \frac{64T_f^{\text{US}}}{W^3}
\]

Due to constant returns to scale, the rent earned by landowners can be calculated as either the value of output, less payments to labor, or as the marginal product of land (the two give the same answer):

\[
\text{US land rent/unit} = \frac{Q_f^{\text{US}} - WL_f^{\text{US}}}{T_f^{\text{US}}} = 2 \left( \frac{L_f^{\text{US}}}{T_f^{\text{US}}} \right)^{2/3} = \frac{\partial Q_f^{\text{US}}}{\partial T_f^{\text{US}}}
\]

(a) Show how increases in labor supply \( L_f^{\text{US}} \), given \( T_f^{\text{US}} = 200 \), affect the wage rate and the return to land owners (the rent on land).

(b) Let the total domestic U.S. labor force be \( L_f^{\text{US}} = 25 \). Find the equilibrium output, wage rate and rent on land in the US, assuming full employment (so \( L_f^{\text{US}} = L_f^{\text{US}} = 25 \)).

(c) Assume the Mexican wage rate is 4 and that Mexican workers will move to the U.S., if allowed, as long as US wages are at least as high as in Mexico. Let \( I \) denote the number of immigrant workers from Mexico, so the total number of workers in the US is:

\[
L_f^{\text{US}} = L_f^{\text{US}} + I = (25 + I).
\]

i. How does immigration from Mexico affect U.S. wages and the return on land? In the US who will favor immigration and who will oppose it?

ii. US net national income \( Y^n \) is US output less wages paid to Mexican immigrants \( W^m \):

\[
Y^n = Q_f^{\text{US}} - W^m I = 6 \left( 200 \right)^{1/3} \left( 25 + I \right)^{2/3} - W^m I
\]

If Mexican workers are paid the same wage as US workers, how does immigration affect US net income and wages? Explain.

iii. If you allow free immigration between the US and Mexico, what ultimately will happen to the wage difference between the two countries?
(d) Assume all workers in the US – including immigrant workers, receive free health insurance from the US government. If there were no restrictions on immigration, must the US gain (i.e., must US net national income increase) by allowing immigrant workers? Explain.

(e) Finally, suppose the production function for food in Mexico is:

\[ Q_f^m = 3 \left( T_f^m \right)^{1/3} \left( L_f^m \right)^{2/3} \]

where \( T_f^m = 100 \), is the amount of fixed land in Mexico, \( L_f^m \) is the number of workers employed in Mexico and the Mexican population is \( L^m = 25 \).

i. Let \( I \) denote the number of Mexican workers who move to the US, so that the work force in each country is:

\[ L_f^m = (L_f^{us} + I) = (25 + I); \quad L_f^m = (L_f^m - I) = (25 - I) \]

Starting from \( I = 0 \), how does immigration from Mexico to the US affect total world output \( (Q_f^{us} + Q_f^m) \)?

ii. Calculate the level of immigration that maximizes total world output.

3. (Chapter 8, Trade Policy) Consider a small country (Thailand) with the following demand and supply curves for steel:

Supply = \( 7(P_s - 100) \); Demand = \( 1800 - 3P_s \)

(output is zero if price is below 100). Assume Thailand can export steel at a given world price of \( P_s = 500 \). Further, assume that Thailand imposes an export tariff of \( t \) per unit of export.

(a) Show how: domestic price, consumption and production change as \( t \) increases. Also, calculate how consumer surplus, producer surplus, and government tariff revenue change as \( t \) increases. What happens to overall welfare in Thailand as \( t \) increases?

i. If \( t > 250 \), what happens to the level of trade?

(b) Compare the domestic equilibrium when \( t = 150 \) to the case in which there is an export quota of 1,000 units (but no tariff). How do exports, domestic price, production and consumption compare under the two plans? What happens to the tariff revenue? Which policy is better for the country?

(c) Suppose, instead of an export tax (tariff), the Thai government subsidized exports at a rate of \( s \) per unit of export. Thus, for each unit of steel exported, the exporter receives \( \{500 + s\} \): 500 comes from the world market, and \( s \) comes from the government. Show how this export subsidy affects:

(i) domestic price; (ii) consumer surplus; (iii) producer surplus; (iv) government expenditures on the subsidy; and (v) overall welfare.

i. Is there any export quota that would have the same effect as the export subsidy?

ii. If the export tariff lowers welfare, can we conclude the export subsidy raises welfare? Explain.
4. (5 points extra credit) (More sophisticated version of H-O model). As in question #1, suppose there are two goods (C and F) and two inputs (K and L). However, the technology for producing each good now allows smooth substitutability among the inputs. The production functions are:

\[ Q_c = K_c^{1/4} L_c^{3/4}, \quad Q_f = K_f^{3/4} L_f^{1/4} \]

where \( \{K_c, L_c\} \) are the inputs (capital, labor) used in sector C and \( \{K_f, L_f\} \) are the inputs used in sector F. Let \( W \) denote the wage rate (price of L) and \( R \) the rental rate (cost of using K, capital). Finally, let \( P_c, P_f \) denote the output prices of goods C and F, respectively.

(a) Derive the cost function for each good by minimizing cost for a given output level. Which good is capital intensive? Why? (Procedure for good C: Minimize \( WL + RK \) subject to the constraint \( Q_c = K_c^{1/4} L_c^{3/4} \), which implies \( K_c = (Q_c^4 L_c^{-3}) \). Use a similar setup for good F).

(b) Given output prices, show how an increase in the supply of capital changes output of each good. Use your result to predict which good the capital-abundant country will export.

i. Given output prices, does the increase in input (capital) supply change factor prices? Explain.

(c) Assuming both goods are produced (so that Price=Marginal Cost for each good), show how an increase in \( P_c \) will affect factor prices (\( W, R \)). Does either factor price rise proportionally more than \( P_c \)? If so, explain which one and why.

(d) Use your answer to parts (b) and (c) to explain how trade will affect the distribution of income in a capital-abundant country (like the US). Will everybody gain from trade? Explain.

(e) Assume the U.S. is capital-abundant. Which group in the U.S. is likely to favor import tariffs and which group is likely to impose trade restrictions? Explain.